Convexity of Trend following & Variance arbitrage

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Convexity of CTA

Outlook

1. Trend and Convexity
2. Trend versus Risk Parity
3. Variance arbitrage
4. Conclusions
How to observe the convexity of CTA?

- Monthly returns of the Barclays BTOP 50 Index vs. monthly returns of the S&P 500 Index (symbols) and parabolic fit

- Monthly returns of the SG CTA Index vs. monthly returns of the S&P 500 Index (symbols) and parabolic fit
Approach: Volatility at different timescale

- Price model

\[ S_t = S_0 + \sum_{t'=1}^{t} D_{t'} . \]

- Price changes \( D_{t'} \) are stationary random variables with zero mean and covariance given by:

\[ \mathbb{E}[D_u D_v] = C(|u - v|) , \]

- Uncorrelated random walks corresponds to \( C(u) = \sigma^2 \delta_{u,0} \). Trending random walks are such that \( C(u) > 0 \), while mean-reverting random walks are such that \( C(u) < 0 \).

- How does this translate in terms of the volatility of the walk? We define the volatility of scale \( \tau \):

\[ \sigma^2(\tau) := \frac{1}{\tau} \mathbb{E} \left[ (S_{t+\tau} - S_t)^2 \right] = \sigma^2 + \frac{2}{\tau} \sum_{u=1}^{\tau} (\tau - u) C(u) , \]

with single step volatility \( \sigma^2(1) = \sigma^2 \).
Simple Trend = long variance - short variance

Consider a simple strategy such that the position $\Pi_t$ is proportional to the price difference between $t$ and 0:

$$\Pi_t := (S_t - S_0),$$

P&L from $t - 1$ to $t$ is given by:

$$G_t := \Pi_{t-1} D_t = D_t \sum_{t'=1}^{t-1} D_{t'}, \quad G_1 := 0.$$

Cumulative performance of from day 0 to day $T$:

$$G_T = \sum_{t=1}^{T} G_t = \sum_{t=2}^{T} \sum_{t'=1}^{t-1} D_t D_{t'},$$

$$= \frac{1}{2} \left( \sum_{t=1}^{T} D_t \right)^2 - \frac{1}{2} \sum_{t=1}^{T} D_t^2 \Rightarrow G_T = \frac{1}{2} \left( S_T - S_0 \right)^2 - \frac{1}{2} \sum_{t=1}^{T} D_t^2,$$

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Convexity with general trend

- Trend estimator defined with EMA filter:
  \[ d\pi_t = -\frac{2}{\tau} \pi_t dt + \frac{2}{\tau} dS_t. \]

The solution of this SDE given by the following expression:
\[ \pi_t = L_\tau(S_t) = \frac{2}{\tau} \int_{-\infty}^{t} e^{2(t-s)/\tau} dS_s. \]

- Daily profit and loss: \( dG_t = \phi(\pi_t) \times dS_t \)
  \[ dG_t = \phi(\pi_t) \pi_t dt + \frac{\tau}{2} \phi(\pi_t) d\pi_t. \]

Let \( F(x) \) be such that \( F'(x) = \phi(x) \), then using Ito’s lemma we have:
\[ dF(\pi_t) = \phi(\pi_t) dP_t + \frac{2\phi'(\pi_t)}{\tau^2} dS_t^2. \]

Inserting this expression in the equation of the P&L, we obtain:
\[ dG_t = \phi(\pi_t) \pi_t dt + \frac{\phi'(\pi_t)}{\tau} dS_t^2 + \frac{\tau}{2} dF(\pi_t) \]

Re-arrange the different terms and introduce the filter \( L_T \):
\[ L_T[dG_t] = \frac{\tau}{T} F(\pi_t) - L_T \left[ \frac{\phi'(\pi_t)}{\tau} dS_t^2 \right] + L_T \left[ \phi(\pi_t) \pi_t - \frac{\tau}{T} F(\pi_t) \right] dt \]
Illustration of convexity

- Linear trend: $\mathcal{L}_{\tau/2}[dG_t] = \pi_t^2 - \frac{1}{\tau} \mathcal{L}_{\tau/2}[dS_t^2]$.  

- Sign of the trend: $\mathbb{E} \mathcal{L}_{\tau}[dG_t | \pi_t] = |\pi_t| - \sqrt{\frac{2}{\tau}} \sigma$. 

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Replication of Risk Parity and CTA

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<tr>
<th>Commodity Sector</th>
<th>Stock Indices</th>
<th>Foreign Exchange Rates</th>
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<td>S&amp;P 500 (CME)</td>
<td>EUR/USD (CME)</td>
<td>Euribor (ICE)</td>
<td>10Y U.S. Treasury Note (CME)</td>
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<td>Gold (CME)</td>
<td>EuroStoxx 50 (Eurex)</td>
<td>JPY/USD (CME)</td>
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<td>Copper (CME)</td>
<td>FTSE 100 (ICE)</td>
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<td>Short Sterling (ICE)</td>
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<td>Soybean (CME)</td>
<td>Nikkei 225 (JPX)</td>
<td>AUD/USD (CME)</td>
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<td>JGB (JPX)</td>
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- Employ the most liquid futures in each sector.
- This selection is stable across time.
- The time series are considered between January 2000 and October 2015.
Correlation between CTA replicator and the SG CTA Index as a function of the time-scale of the trend. Maximum is around $\tau = 180$ days.

Cumulated returns of trend replicator and the SG CTA Index. We seem to capture all the alpha contained in the SG CTA Index.
Convexity of CTA versus S&P500

- Plot aggregated performance over \( \tau' \) days the SG CTA Index as a function of a \( \tau \)-day trend on S&P 500 index.
- We need a careful choice of timescale to observe the convexity. Trend on S&P 500 is computed with \( \tau = 180 \) while trend on SG CTA Index is computed with \( \tau' \approx 90 \).
- Convexity feature is much more significantly than the first monthly plot.
A simple trend following strategy applied on Risk Parity provides the exact quadratic behavior of convexity.

CTA index provides also convexity to Risk Parity strategy.

We can derive an inequality for lower bound of convexity:

$$E[G_{CTA} | T_{RP}] \geq \gamma(\tau) \left( (T_{RP})^2 - 1 \right)$$

where $T_{RP}$ is the trend on Risk Parity index.
Conclusion: trend as a hedge of Risk Parity

- Trend following is a natural way to do stop-loss for long risk parity.
- The hidden cost of for doing this hedge is the realized volatility.
- Tuning the mixing between Trend and Long-only, one may obtain the desired the convexity for the portfolio.
- Both global trend and diversified trend can be used to hedge long risk parity.
Straddle and collection of strangles

- ATM straddle payoff:

\[ G_T^{\text{straddle}} := |S_T - S_0| - (C_{0,T}^{S_0} + P_{0,T}^{S_0}) \]
\[ = |S_T - S_0| - \sqrt{\frac{2T}{\pi}} S_0 \sigma_a^{0} \]

- For a collection of strangles:

\[ G_T^{\text{strangles}} := \int_0^{S_0} (K - S_T)^+ dK + \int_{S_0}^{\infty} (S_T - K)^+ dK - \int_0^{S_0} P_{0,T}^{K} + \int_{S_0}^{\infty} C_{0,T}^{K} \]
\[ \frac{1}{2} (S_T - S_0)^2 - \frac{1}{2} T \sigma_0^{2} \]

We obtain the payoff:

\[ G_T^{\text{strangles}} = \frac{1}{2} (S_T - S_0)^2 - \frac{1}{2} T \sigma_0^{2} \]

Buying straddle or strangles is a simple way to have long exposure on long-term variance by paying fixed implied volatility.
Trend as Delta-hedge: Model-free approach

- When the price moves, we need to trade a quantity of underlying to keep neutral Delta. The Delta-hedge position is given simply by:

\[ \Delta_{hdg} = -(S_t - S_0) \]

- Total P&L of the hedge from \( t = 0 \) to the maturity is given by:

\[ G_{hedge}^T = \frac{1}{2} \sum_{t=1}^{T} D_t^2 - \frac{1}{2} (S_T - S_0)^2 \]

- Add the pay-off of strangles to this hedge P&L, we find:

\[ G_{strangles}^T + G_{hedge}^T = \frac{1}{2} \sum_{t=1}^{T} D_t^2 - \frac{T}{2} \bar{\sigma}_{0,T}^2 \]

Variance Swap payoff

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Variance arbitrage

- Selling option will allow a risk premium. In fact, $\mathbb{E}[G^\text{strangles}_T] < 0$ because the implied volatility is usually overpriced and more expensive than the realized volatility.

\[
G^\text{strangles}_T = \frac{1}{2} (S_T - S_0)^2 - \frac{1}{2} T \bar{\sigma}_{0,T}^2
\]

- Doing simple trend is also a simple way to exposure to long-variance. Indeed, trend anomaly has been proved empirically $\mathbb{E}[G^\text{trend}_T] > 0$.

\[
G^\text{trend}_T = \frac{1}{2} (S_T - S_0)^2 - \frac{1}{2} \sum_{t=1}^{T} D_t^2
\]

- Trading volatility is interesting because we profit from both effects. Selling long-variance with high cost "implied variance" and buying long-variance with smaller cost "realized variance" $\Rightarrow$ Variance arbitrage

\[
G^\text{trend}_T - G^\text{strangles}_T = \frac{1}{2} T \bar{\sigma}_{0,T}^2 - \frac{1}{2} \sum_{t=1}^{T} D_t^2
\]
Conclusions

Trend following and convexity:
- Trend is an arbitrage between long-term variance and short-term variance
- Convexity of trend is observed in association with a timescale (investment horizon or trend timescale)
- Timescale of trend defines a maturity (like call/put). Trend behaves as a hedge only for this maturity.

Trend following as a hedge of Risk Parity:
- Trend can be used as a hedging tool of long risk parity
- Both diversified trend and global trend can be used for hedging the long risk

Variance arbitrage:
- Delta hedge of options is a trend. To harvest option premium, hedging frequency is important.
- Choosing hedging frequency is choosing timescale of volatility that one wants to be exposed.
- Best choice for hedging is to employ the trend anomaly with low frequency.